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COMPUTER SIMULATION OF PLANE CURVES IN RESPECT OF THEORETICAL DRAWING OF THE SHIP'S HULL

A new approach to modeling of planar curves with linear curvature distribution in dependence of the arc length is discussed. The simulated curve is given in tabular form. At the start and end points of curve the angles of tangents are specified. A computer code of calculation and visualization of the results on a PC was developed.

Keywords: flat curve, computer modeling, linear dependence of the distribution of curvature, the arc length.

Widespread use of curves in science and technology, new areas of application encourage specialists in applied geometry to develop new approaches to modeling lines, adapting them to the needs of the practice. Particular importance shall modeling lines in the design drawing theoretical hull. Curves describing the waterline frames and batoksy and received interpolation control points must be invariant with respect to the rotation of the coordinate system and depend on the location of control points. At the same time the curve and its first derivative must be continuous and curvature lines obtained by interpolation, has the largest degree determined by linear dependence of its length and being continuous function. Thus, the theoretical drawing lines modeling the hull at the aggregate reference points necessary to provide a second order of smoothness.

Considering known in mathematics curves, we can conclude that the set above conditions may meet only modified clothoid.

In practice, the construction of shipbuilding theoretical hull drawings often use so-called simple interpolator [7] which satisfy the conditions of continuity as reference points function and its first derivative. These applied parametric polynomials of degree not lower than third. The downside polynomial submission hull lines is the presence of extreme value relative curvature curve between two reference points. In particular, for the curves with a small change in curvature, typical local "bulging" when reference points are not close to each other.

Recently in modeling curves started to apply parametric equation, which serves the arc length parameter of the curve. This means that for these curves can be found in the function of the equation arc length. Unfortunately, these curves are very few and not all are suitable for practical applications because the authors of [1-6, 8-10] in modeling curves that meet certain conditions have to take into consideration linear, quadratic, cubic, sinusoidal or other relationship between the curvature of the arc length and determine the unknown coefficients of these relationships in the simulation curve that meets the conditions set design of an object or product.

The authors of [2, 4, 10] propose to model curves using linear distribution laws of curvature, taking them as:

$$k = as + b,$$

where s – the arc length of the curve, a and b – unknown coefficients.

These coefficients are determined, provided that certain start and end points of the curve and angles in these tangents.

The aim is to design and research interpolation plane curves, which are given a certain amount of control points, providing a docking point second order smoothness and curvature of the curve linearly depends on the length of the arc. The problem is solved, provided that specified angles tangential to the start and end points of the curve. Simulated curves must be suitable for the construction of the theoretical drawing of hull.

Construct an arbitrary curve, given a sequence known points and angles pertaining to the first and last points. Fig. 1 shows the starting points are connected by a broken line, which can be seen as a zero-order interpolation curve smoothness. At the start and end points made pieces of straight lines, angles are set with the original data. It is necessary for a given output data interpolation to build the curve of the second order smoothness.

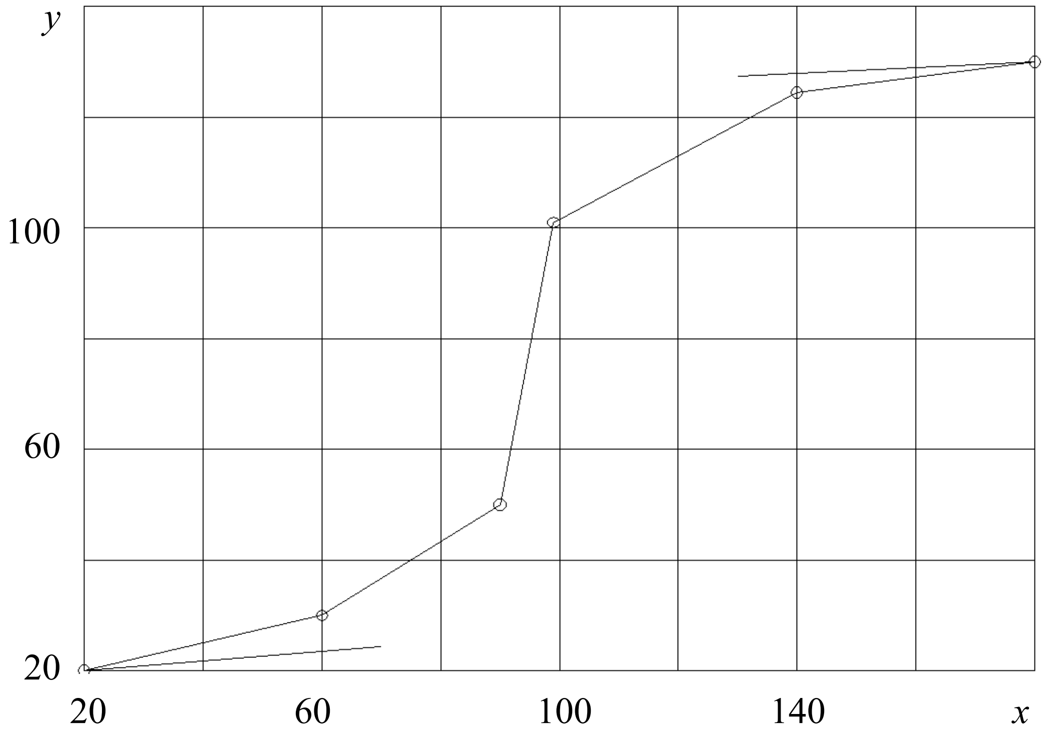

Fig. 1. Input data to build the interpolation curve

Consider the plot curve defined two consecutive reference points x_i, y_i and x_{i+1}, y_{i+1}. Tangent to the curve at these points have angles to the x-axis, respectively, α_i and α_{i+1}.

We can write the equation as follows clothoid

$$x(s) = x_i + b_i \int_{-1}^{s} \cos \Phi_i(s) ds,$$
$$y(s) = y_i + b_i \int_{-1}^{s} \sin \Phi_i(s) ds, \quad (1)$$

where

$$\Phi_i(s) = \varphi_i - \psi_i s + \gamma_i (s^2 - 1).$$

At the same angles φ_i and ψ_i are defined by expressions

$$\varphi_i = \frac{1}{2}(\alpha_i + \alpha_{i+1});$$
$$\psi_i = \frac{1}{2}(\alpha_i - \alpha_{i+1}),$$

i. e, they depend on the relevant angles in adjacent locations.

For construction sites curve between points x_i, y_i and x_{i+1}, y_{i+1} must find constants b_i and γ_i. To determine these constants consider section curve in the coordinate system $\bar{x}$, $\bar{y}$ with the origin at the point x_i, y_i,, which rotates at an angle η_i, which is the angle to the horizontal axis segment of the straight line connecting and i and $(i + 1)$ points of the simulated curve.

It's easy to see that when executed with the following conditions:

$$\bar{y}(1) = 0 = b_i \int_{-1}^{1} \cos(\Phi_i(s) - \eta_i) ds; \qquad (2)$$

$$\bar{x}(1) = d_i = b_i \int_{-1}^{1} \sin(\Phi_i(s) - \eta_i) ds, \qquad (3)$$

where

$$d_i = \sqrt{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2}.$$

Since d_i is not zero, then b_i can not be zero. In this regard, equation (2) can be used with constant iterative determination η_i. B_i constant determined from equation (3). Modern computer technology allows numerical methods to determine the constants b_i and η_i without any problems.

For the calculation of the expressions (1) Cartesian coordinates of the curve needs to know the angles α_i and α_{i+1} tilt in tangential reference points. These corners will also determine the numerical method. In the first approach angle, which will start the process of search is invited angles are equal angles segments connecting the previous and next supporting point (fig. 2). This figure shows that the tangent to the curve in the future reference points are parallel segments that connect the $(i - 1)$ and $(i + 1)$ reference points.

Fig. 3 shows the interpolation curve constructed using tilt angles of tangents at above a definite pattern. These results can be seen as the source curve interpolation approach. At first glance, this curve is acceptable character. But a more careful consideration it can be concluded that it is desirable to improve the character of the passage of the curve at the first, middle and last part of it. This is a consequence of what happened curve modeling without taking into account the fact that the reference points of this differential characteristic calculated at the end of the previous and early next section should be the same.

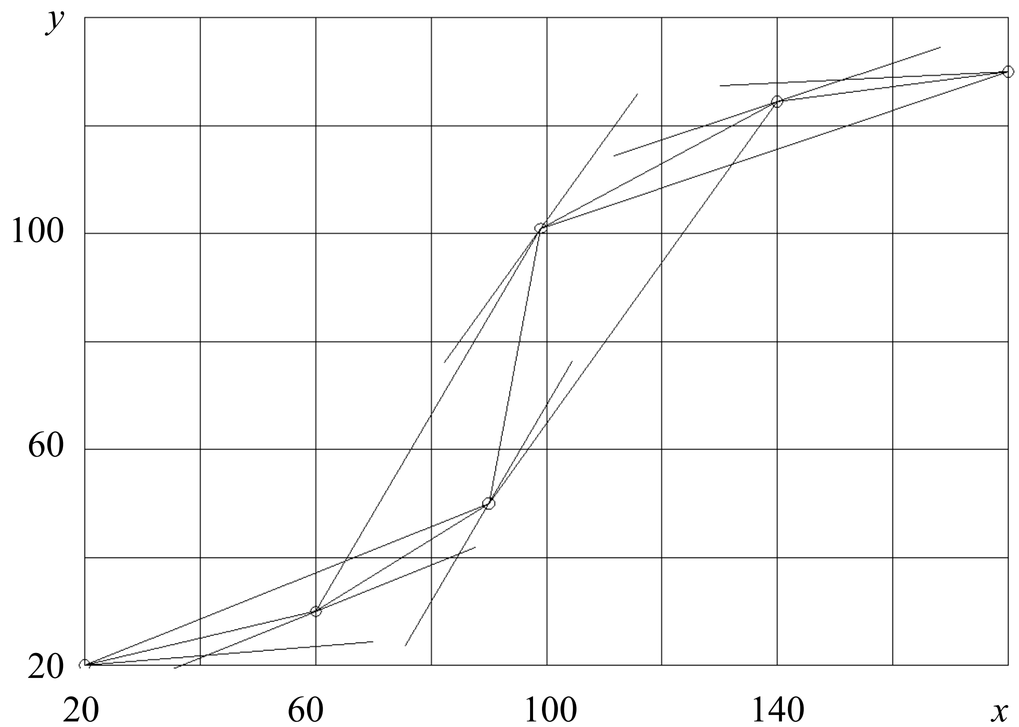

Fig. 2. To determine the angles of tangents at intermediate points

For parametric curve defined by equations (1) the curvature can be determined by dependence

$$k_i(s) = \frac{x'(s)y''(s) - y'(s)x''(s)}{(x'^2 + y'^2)^{3/2}} = \frac{2\gamma_i s - \psi_i}{b_i}. \qquad (4)$$

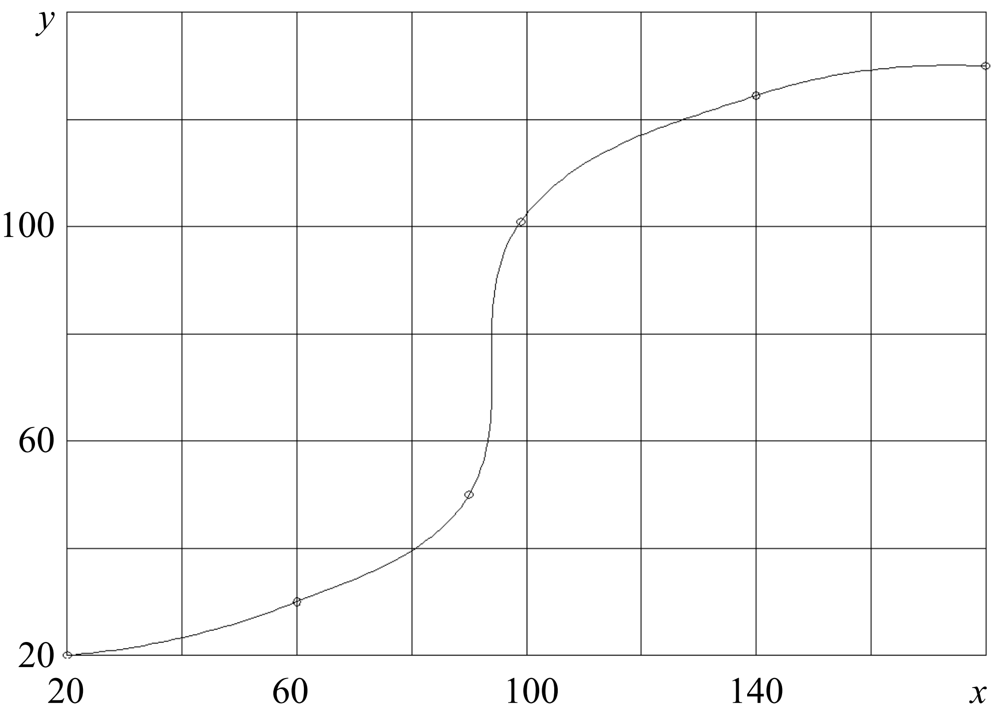

Fig. 3. The interpolation curve of the first approach

One can easily see that the curvature of the curve, which is based on dependencies (1) is a linear function of the parameter *s*. Indeed, the arc length is the expression

$$S_i(s) = \int_{-1}^{s} \sqrt{x'^2 + y'^2}\, ds = (1+s)b_i. \tag{5}$$

From the expressions (4) and (5) that the arc length is a linear function of the parameter s, and at the same time, the curvature is increased or decreased in proportion to its length.

In table 1 shows the values of curvature curve interpolation initial approach to the intermediate reference points defined by dependence (4).

Table 1

The value of curvature at reference points

Point number	$k_i(s=1)$	$k_{i+1}(s=-1)$
1	$7{,}830416\times10^{-3}$	$-6{,}029317\times10^{-3}$
2	$4{,}208951\times10^{-2}$	$4{,}074901\times10^{-2}$
3	$-4{,}412976\times10^{-2}$	$-3{,}068910\times10^{-2}$
4	$4{,}279898\times10^{-3}$	$-1{,}108483\times10^{-3}$

Note. In table 1 numbered only intermediate reference points. The initial reference point has zero number.

From the analysis of table 1 shows that in all the reference points curvature area on the left side of the reference point, no curvature plot is located right in relation to the reference point. This means that the output interpolation approach is actually a curve with the first order of smoothness in the key points, which reached parity values of functions and their derivatives, and suffers curvature gap.

Achieving equality curvature of the left and right of the control points by minimizing functional realize the following form

$$f = \sum_{i=1}^{n} \left(k_{i(s=1)} - k_{i+1(s=-1)}\right)^2, \tag{6}$$

where $n = 4$ (in the case in question).

In functional minimization problem recorded for options that range are taken tangential angles at the nodal points of the above approach them first.

To minimize the functional (6) used high-performance algorithm proposed Hook-Jeeves [11] and is designed to minimize the function of many variables.

Solving optimization problem related to the definition of angles relevant to the reference point align allowed at these points mentioned curvature taken in areas left and right of the control points (table 2).

Table 2

Optimal values of curvature as reference points.

Point number	$k_i(s=1)$	$k_{i+1}(s=-1)$
1	$1{,}608085\times10^{-3}$	$1{,}608085\times10^{-3}$
2	$3{,}721636\times10^{-2}$	$3{,}721636\times10^{-2}$
3	$-3{,}549732\times10^{-2}$	$-3{,}549730\times10^{-2}$
4	$-7{,}863768\times10^{-5}$	$-7{,}863050\times10^{-5}$

As the review of the table, the values of curvature at nodal points consistent with a sufficiently high degree of accuracy. This image also confirms the results presented in fig. 4, which shows the comparison output interpolation curve (solid line) and curve constructed with optimal values of tilt angles relevant to the reference points (dashed line).

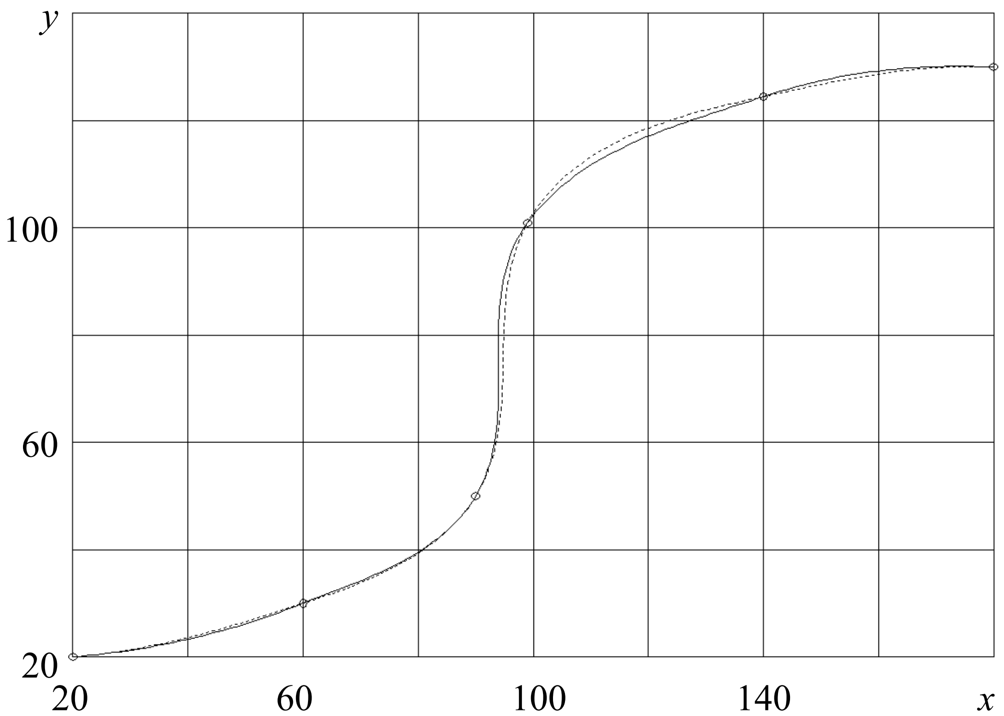

Fig. 4. The results of the interpolation curve modelling

Through modeling interpolation curve with optimal values of tilt angles relevant reference points in the character of its passage substantially improved.

Fig. 5 to confirm the applicability of the developed method of modeling the curve interpolation to build a theoretical drawing shows the hull fragment, which depicts one of waterline cargo ship. Net drawings diametric plane formed by the projection (DP) to the main plane tangent bottom shell, two and twenty one buttocks theoretical frames.

The practical application of proven possibility of modeling curves using an improved method for determining the linear dependence of curvature of the arc length. The method can be applied

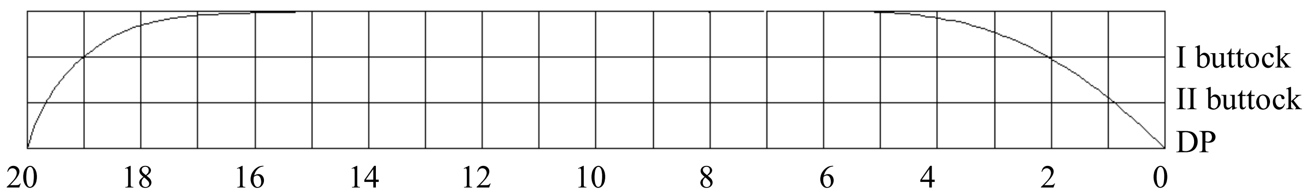

Fig. 5. Fragment of theoretical drawing of hull

when simulating ship curves. Further efforts in the modeling interpolation lines can be directed to ensure the application of the laws of distribution of higher degrees of curvature.

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КОМП'ЮТЕРНЕ МОДЕЛЮВАННЯ ПЛОСКИХ КРИВИХ СТОСОВНО ДО ТЕОРЕТИЧНОГО КРЕСЛЕННЯ КОРПУСУ СУДНА

В статті розглядається новий підхід до моделювання плоских кривих із застосуванням лінійної залежності розподілу кривини від довжини дуги. Крива, що моделюється задається в табличній формі. В початковій і кінцевій точках кривої визначаються кути нахилу дотичних. Розроблено програму розрахунків і візуалізації отриманих результатів на ПЕОМ.

Ключові слова: плоска крива, комп'ютерне моделювання, лінійна залежність розподілу кривини, довжина дуги.

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КОМПЬЮТЕРНОЕ МОДЕЛИРОВАНИЕ ПЛОСКИХ КРИВЫХ ПРИМЕНИТЕЛЬНО К ТЕОРЕТИЧЕСКОМУ ЧЕРТЕЖУ КОРПУСА СУДНА

В статье рассматривается новый подход к моделированию плоских кривых с применением линейной зависимости распределения кривизны от длины дуги. Моделируемая кривая задается в табличной форме. В начальной и конечной точках кривой заданны углы наклона касательных. Разработана программа расчетов и визуализации полученных результатов на ПЭВМ.

Ключевые слова: плоская кривая, компьютерное моделирование, линейная зависимость распределения кривизны, длина дуги.

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